Brazil National Olympiad 2008

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Day 1

1 A positive integer is *dapper* if at least one of its multiples begins with 2008. For example, 7 is dapper because 200858 is a multiple of 7 and begins with 2008. Observe that $200858 = 28694 \times 7$.

Prove that every positive integer is dapper.

2 Let S be a set of $6n$ points in a line. Choose randomly $4n$ of these points and paint them blue; the other $2n$ points are painted green. Prove that there exists a line segment that contains exactly $3n$ points from S, $2n$ of them blue and n of them green.

3 Let x, y, z real numbers such that $x + y + z = xy + yz + zx$. Find the minimum value of

$$\frac{x}{x^2+1} + \frac{y}{y^2+1} + \frac{z}{z^2+1}$$

Day 2

1 Let $ABCD$ be a cyclic quadrilateral and r and s the lines obtained reflecting AB with respect to the internal bisectors of $\angle CAD$ and $\angle CBD$, respectively. If P is the intersection of r and s and O is the center of the circumscribed circle of $ABCD$, prove that OP is perpendicular to CD.

2 Prove that for all integers $a > 1$ and $b > 1$ there exists a function f from the positive integers to the positive integers such that $f(a \cdot f(n)) = b \cdot n$ for all n positive integer.

3 The venusian prophet Zabruberson sent to his pupils a 10000-letter word, each letter being A or E: the *Zabrubic word*. Their pupils consider then that for $1 \leq k \leq 10000$, each word comprised of k consecutive letters of the Zabrubic word is a *prophetic word* of length k. It is known that there are at most 7 prophetic words of lenght 3. Find the maximum number of prophetic words of length 10.

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