**The senior division problems of the 2014 Iranian Geometry Olympiad.**

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by YESMAths, MRF2017

**1:** ATimo : ABC is a triangle with A=90 and C=30.Let M be the midpoint of BC. Let W be a circle passing through A tangent in M to BC. Let P be the circumcircle of ABC. W is intersecting AC in N and P in M. prove that MN is perpendicular to BC.

Let me correct your question and LaTeXify it. :)

__Question__ $ABC$ is a triangle with $\angle BAC = 90^\circ$ and $\angle ACB = 30^\circ$. Let $M_1$ be the midpoint of $BC$. Let $W$ be a circle passing through $A$ tangent in $M_1$ to $BC$. Let $P$ be the circumcircle of $ABC$. $W$ is intersecting $AC$ in $N$ and $P$ in $M$. Prove that $MN$ is perpendicular to $BC$.

__Solution__
Let $M_1$ be the midpoint of $BC$. And let $C_1$ be the center of $W$. So, we first show that $C_1 \in AC$.
Clearly $C_1M_1 \perp BC$. And also, since $M_1$ is the center of $P$ as $ABC$ is a right triangle, we have $M_1A = M_1B = M_1C$.
So, $\angle M_1AB = \angle M_1BA = 60^\circ$. And this implies that $\triangle AM_1B$ is equilateral. So, $\angle C_1M_1A = 90^\circ - 60^\circ = 30^\circ = \angle C_1AM_1$.
But then, $\angle C_1AM_1 + \angle M_1AB = 90^\circ = \angle C_1AB = \angle CAB$. This shows that $\boxed{C_1 \in AC}$.
Next $C_1N = C_1M_1$ and $\angle AC_1M_1 = 180^\circ - 30^\circ - 30^\circ = 120^\circ = 180^\circ - \angle NC_1M_1$, showing that $\triangle NC_1M_1$ is equilateral.
Again, $C_1M = C_1A$ showing $\angle C_1AM = \angle C_1MA$.
Now note that $AM$ is the radical axis of $P$ and $W$. So, $M_1C_1 \perp AM$. Let $M_1C_1 \cap AM = X$.
Then, $AC_1X = 60^\circ \Rightarrow NC_1M = 180^\circ - \angle MC_1X - 60^\circ = 60^\circ$.
But $C_1M = C_1N$. Thus $\angle MNC_1 = \angle NC_1M \Rightarrow MN \parallel M_1C_1 \Rightarrow \boxed{MN \perp BC}$.
This completes the proof. :)

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**2:** In the Quadrilateral $ABCD$ we have $\measuredangle B = \measuredangle D = 60^\circ$.$M$ is midpoint of side $AD$.The line through $M$ parallel to $CD$ meets $BC$ at $P$.Point $X$ lying on $CD$ such that $BX = MX$.Prove that $AB = BP$ if and only if $\measuredangle MXB = 60^\circ$.

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**3:** Let $ABC$ be an acute triangle.A circle with diameter $BC$ meets $AB$ and $AC$ at $E$ and $F$,respectively. $M$ is midpoint of $BC$ and $P$ is point of intersection $AM$ with $EF$. $X$ is an arbitary point on arc $EF$ and $Y$ is the second intersection of $XP$ with a circle with diameter $BC$.Prove that $\measuredangle XAY = \measuredangle XYM$.
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**4:** A tangent line to circumcircle of acute triangle $ABC$ ($AC > AB$) at $A$ intersects with the extension of $BC$ at $P$. $O$ is the circumcenter of triangle $ABC$.Point $X$ lying on $OP$ such that $\measuredangle AXP = 90^\circ$.Points $E$ and $F$ lying on $AB$ and $AC$,respectively,and they are in one side of line $OP$ such that $\measuredangle EXP = \measuredangle ACX$ and $\measuredangle FXO = \measuredangle ABX$. $K$,$L$ are points of intersection $EF$ with circumcircle of triangle $ABC$.prove that $OP$ is tangent to circumcircle of triangle $KLX$.

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**5:** Two points $P$ and $Q$ lying on side $BC$ of triangle $ABC$ and their distance from the midpoint of $BC$ are equal.The perpendiculars from $P$ and $Q$ to $BC$ intersect $AC$ and $AB$ at $E$ and $F$,respectively.$M$ is point of intersection $PF$ and $EQ$.If $H_1$ and $H_2$ be the orthocenters of triangles $BFP$ and $CEQ$, respectively, prove that $AM \perp H_1H_2$.

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