Greece National Olympiad 1995

www.artofproblemsolving.com/community/c5175

by socrates, M4RI0, matex (L)(L)(L)

1 Find all positive integers n such that $-5^4+5^5+5^n$ is a perfect square. Do the same for $2^4+2^7+2^n$.

---

2 Let ABC be a triangle with $AB=AC$ and let D be a point on BC such that the incircle of ABD and the excircle of ADC corresponding to A have the same radius. Prove that this radius is equal to one quarter of the altitude from B of triangle ABC.

---

3 If the equation $ax^2+(c-b)x+(e-d)=0$ has real roots greater than 1, prove that the equation $ax^4+bx^3+cx^2+dx+e=0$ has at least one real root.

---

4 Given are the lines $l_1, l_2, \ldots, l_k$ in the plane, no two of which are parallel and no three of which are concurrent. For which k can one label the intersection points of these lines by $1, 2, \ldots, k-1$ so that in each of the given lines all the labels appear exactly once?

Art of Problem Solving is an ACS WASC Accredited School.