Greece National Olympiad 1997

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by socrates

1 Let P be a point inside or on the boundary of a square $ABCD$. Find the minimum and maximum values of $f(P)=\angle ABP+\angle BCP+\angle CDP+\angle DAP$.

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2 Let a function $f:\mathbb{R}^+\to\mathbb{R}$ satisfy:
(i) f is strictly increasing,
(ii) $f(x)>-1/x$ for all $x>0$,
(iii)$f(x)f(f(x)+1/x)=1$ for all $x>0$.
Determine $f(1)$.

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3 Find all integer solutions to

$$\frac{13}{x^2}+\frac{1996}{y^2}=\frac{z}{1997}.$$

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4 A polynomial P with integer coefficients has at least 13 distinct integer roots. Prove that if an integer n is not a root of P, then $|P(n)|\geq 7\cdot 6!^2$, and give an example for equality.
