Greece National Olympiad 1999

1   Let $f(x) = ax^2 + bx + c$, where a, b, c are nonnegative real numbers, not all equal to zero. Prove that $f(xy)^2 \leq f(x^2)f(y^2)$ for all real numbers x, y.

2   A right triangle has integer side lengths, and the sum of its area and the length of one of its legs equals 75. Find the side lengths of the triangle.

3   In an acute-angled triangle ABC, AD, BE and CF are the altitudes and H the orthocentre. Lines EF and BC meet at N. The line passing through D and parallel to FE meets lines AB and AC at K and L, respectively. Prove that the circumcircle of the triangle NKL bisects the side BC.

4   On a circle are given $n \geq 3$ points. At most, how many parts can the segments with the endpoints at these n points divide the interior of the circle into?