Greece National Olympiad 2000

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1 Consider a rectangle $ABCD$ with $AB = a$ and $AD = b.$ Let l be a line through $O,$ the center of the rectangle, that cuts AD in E such that $AE/ED = 1/2$. Let M be any point on $l,$ interior to the rectangle.
Find the necessary and sucient condition on a and b that the four distances from M to lines AD, AB, DC, BC in this order form an arithmetic progression.

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2 Find all prime numbers p such that $1 + p + p^2 + p^3 + p^4$ is a perfect square.

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3 Find the maximum value of k such that

$$\frac{xy}{\sqrt{(x^2+y^2)(3x^2+y^2)}} \leq \frac{1}{k}$$

holds for all positive numbers x and y.

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4 The subsets $A_1, A_2, \ldots, A_{2000}$ of a finite set M satisfy $|A_i| > \frac{2}{3}|M|$ for each $i = 1, 2, \ldots, 2000.$ Prove that there exists $m \in M$ which belongs to at least 1334 of the subsets A_i.

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