Greece National Olympiad 2002

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–	Seniors
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1	The real numbers a, b, c with $bc \neq 0$ satisfy $\frac{1-c^2}{bc} \geq 0$. Prove that $10(a^2+b^2+c^2-bc^3) \geq 2ab+5ac$.
2	A student of the National Technical University was reading advanced mathematics last summer for 37 days according to the following rules : (a) He was reading at least one hour every day. (b) He was reading an integer number of hours, but not more than 12, each day. (c) He had to read at most 60 hours in total. Prove that there were some successive days during which the student was reading exactly 13 hours in total.
3	In a triangle ABC we have $\angle C > 10^0$ and $\angle B = \angle C + 10^0$.We consider point E on side AB such that $\angle ACE = 10^0$, and point D on side AC such that $\angle DBA = 15^0$. Let $Z \neq A$ be a point of interection of the circumcircles of the triangles ABD and AEC.Prove that $\angle ZBA > \angle ZCA$.
4	(a) Positive integers p, q, r, a satisfy $pq = ra^2$, where r is prime and p, q are relatively prime. Prove that one of the numbers p, q is a perfect square. (b) Examine if there exists a prime p such that $p(2^{p+1}-1)$ is a perfect square.