Greece National Olympiad 2004

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by socrates

1 Find the greatest value of $M \in \mathbb{R}$ such that the following inequality is true $\forall$ $x, y, z \in \mathbb{R}$ $x^4 + y^4 + z^4 + xyz(x + y + z) \geq M(xy + yz + zx)^2$.

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2 If $m \geq 2$ show that there does not exist positive integers $x_1, x_2, ..., x_m$, such that

$$x_1 < x_2 < ... < x_m \text{ and } \frac{1}{x_1^3} + \frac{1}{x_2^3} + ... + \frac{1}{x_m^3} = 1.$$

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3 Consider a circle $K(O, r)$ and a point A outside K. A line ϵ different from AO cuts K at B and C, where B lies between A and C.
Now the symmetric line of ϵ with respect to axis of symmetry the line AO cuts K at E and D, where E lies between A and D.
Show that the diagonals of the quadrilateral $BCDE$ intersect in a fixed point.

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4 Let $M \subset \mathbb{N}^*$ such that $|M| = 2004$.
If no element of M is equal to the sum of any two elements of M,
find the least value that the greatest element of M can take.
