

Greece National Olympiad 2006

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- 1 How many 5 digit positive integers are there such that each of its digits, except for the last one, is greater than or equal to the next digit?
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- 2 Let n be a positive integer. Prove that the equation

$$x + y + \frac{1}{x} + \frac{1}{y} = 3n$$

does not have solutions in positive rational numbers.

- 3 Let a triangle ABC and the cevians AL, BN, CM such that AL is the bisector of angle A . If $\angle ALB = \angle ANM$, prove that $\angle MNL = 90$.
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- 4 Does there exist a function $f : \mathbb{R} \rightarrow \mathbb{R}$, which satisfies both conditions :

a) $f(x + y + z) \leq 3(xy + yz + zx)$ for all real numbers x, y, z

and

b) there exist function g and natural number n , such that

$g(g(x)) = x^{2n+1}$ and $f(g(x)) = (g(x))^2$ for every real number x ?
