Greece National Olympiad 2007

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–	Seniors
1	Find all positive integers n such that $4^n + 2007$ is a perfect square.
2	Let a, b, c be sides of a triangle, show that $$\frac{(c+a-b)^4}{a(a+b-c)} + \frac{(a+b-c)^4}{b(b+c-a)} + \frac{(b+c-a)^4}{c(c+a-b)} \geq ab+bc+ca.$$
3	In a circular ring with radii $11r$ and $9r$, we put circles of radius r which are tangent to the boundary circles and do not overlap. Determine the maximum number of circles that can be put this way. (You may use that $9.94 < \sqrt{99} < 9.95$)
4	Given a 2007×2007 array of numbers 1 and -1, let A_i denote the product of the entries in the ith row, and B_j denote the product of the entries in the jth column. Show that $$A_1 + A_2 + \cdots + A_{2007} + B_1 + B_2 + \cdots + B_{2007} \neq 0.$$