Greece National Olympiad 2008

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by Amir Hossein

1 A computer generates all pairs of real numbers $x, y \in (0,1)$ for which the numbers $a = x + my$ and $b = y + mx$ are both integers, where m is a given positive integer. Finding one such pair (x, y) takes 5 seconds. Find m if the computer needs 595 seconds to find all possible ordered pairs (x, y).

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2 Find all integers x and prime numbers p satisfying $x^8 + 2^{2^x+2} = p$.

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3 A triangle ABC with orthocenter H is inscribed in a circle with center K and radius 1, where the angles at B and C are non-obtuse. If the lines HK and BC meet at point S such that $SK(SK - SH) = 1$, compute the area of the concave quadrilateral $ABHC$.

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4 If $a_1, a_2, \dots, a_n$ are positive integers and $k = \max\{a_1, \dots, a_n\}$, $t = \min\{a_1, \dots, a_n\}$, prove the inequality

$$\left(\frac{a_1^2 + a_2^2 + \cdots + a_n^2}{a_1 + a_2 + \cdots + a_n}\right)^{\frac{kn}{t}} \geq a_1 a_2 \cdots a_n.$$

When does equality hold?
