Greece National Olympiad 2010

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by stergiu

1 Solve in the integers the diophantine equation

$$x^4 - 6x^2 + 1 = 7 \cdot 2^y.$$

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2 If x, y are positive real numbers with sum $2a$, prove that :

$x^3y^3(x^2+y^2)^2 \leq 4a^{10}$

When does equality hold ?

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3 A triangle ABC is inscribed in a circle $C(O,R)$ and has incenter I. Lines AI, BI, CI meet the circumcircle (O) of triangle ABC at points D, E, F respectively. The circles with diameter ID, IE, IF meet the sides BC, CA, AB at pairs of points $(A_1, A_2), (B_1, B_2), (C_1, C_2)$ respectively.

Prove that the six points $A_1, A_2, B_1, B_2, C_1, C_2$ are concyclic.

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4 On the plane are given $k+n$ distinct lines , where $k>1$ is integer and n is integer as well.Any three of these lines do not pass through the

same point . Among these lines exactly k are parallel and all the other n lines intersect each other.All $k+n$ lines define on the plane a partition

of triangular , polygonic or not bounded regions. Two regions are colled different, if the have not common points

or if they have common points only on their boundary.A regions is called "good" if it contained in a zone between two parallel lines .

If in a such given configuration the minimum number of "good" regionrs is 176 and the maximum number of these regions is 221, find k and n.

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