Greece National Olympiad 2011

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1 Solve in integers the equation

$$x^3y^2\left(2y-x\right)=x^2y^4-36$$

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2 In the Cartesian plane Oxy we consider the points $A_1\left(40,1\right),A_2\left(40,2\right),\ldots,A_{40}\left(40,40\right)$ as well as the segments $OA_1,OA_2,\ldots,OA_{40}$. A point of the Cartesian plane Oxy is called "good", if its coordinates are integers and it is internal of one segment $OA_i, i=1,2,3,\ldots,40$. Additionally, one of the segments $OA_1,OA_2,\ldots,OA_{40}$ is called "good" if it contains a "good" point. Find the number of "good" segments and "good" points.

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3 Let a,b,c be positive real numbers with sum 6. Find the maximum value of

$$S=\sqrt[3]{a^2+2bc}+\sqrt[3]{b^2+2ca}+\sqrt[3]{c^2+2ab}.$$

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4 We consider an acute angled triangle ABC (with $AB<AC$) and its circumcircle $c(O,R)$(with center O and semidiametre R).The altitude AD cuts the circumcircle at the point E ,while the perpedicular bisector (m) of the segment AB,cuts AD at the point $L.BL$ cuts AC at the point M and the circumcircle $c(O,R)$ at the point N.Finally EN cuts the perpedicular bisector (m) at the point Z.Prove that:

$$MZ\perp BC \iff (CA=CB \text{ or } Z\equiv O)$$
