Greece National Olympiad 2012

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by chris!!!

1 Let positive integers p, q with $\gcd(p, q) = 1$ such as $p + q^2 = (n^2 + 1)p^2 + q$. If the parameter n is a positive integer, find all possible couples (p, q).

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2 Find all the non-zero polynomials $P(x), Q(x)$ with real coefficients and the minimum degree,such that for all $x \in \mathbb{R}$:

$$P(x^2) + Q(x) = P(x) + x^5 Q(x)$$

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3 Let an acute-angled triangle ABC with $AB < AC < BC$, inscribed in circle $c(O, R)$. The angle bisector AD meets $c(O, R)$ at K. The circle $c_1(O_1, R_1)$(which passes from A, D and has its center O_1 on OA) meets AB at E and AC at Z. If M, N are the midpoints of ZC and BE respectively, prove that:

a)the lines ZE, DM, KC are concurrent at one point T.

b)the lines ZE, DN, KB are concurrent at one point X.

c)OK is the perpendicular bisector of TX.

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4 The following isosceles trapezoid consists of equal equilateral triangles with side length 1. The side A_1E has length 3 while the larger base A_1A_n has length $n - 1$. Starting from the point A_1 we move along the segments which are oriented to the right and up(obliquely right or left). Calculate (in terms of n or not) the number of all possible paths we can follow, in order to arrive at points B, Γ, Δ, E, if n is an integer greater than 3.

[Need image]
