Greece National Olympiad 2013

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by chris!!!

1 Let the sequence of real numbers $(a_n), n = 1, 2, 3...$ with $a_1 = 2$ and $a_n = \left(\frac{n+1}{n-1}\right)(a_1 + a_2 + ... + a_{n-1}), n \geq 2$.
Find the term a_{2013}.

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2 Solve in integers the following equation:

$$y = 2x^2 + 5xy + 3y^2$$

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3 We define the sets $A_1, A_2, ..., A_{160}$ such that $|A_i| = i$ for all $i = 1, 2, ..., 160$. With the elements of these sets we create new sets $M_1, M_2, ...M_n$ by the following procedure: in the first step we choose some of the sets $A_1, A_2, ..., A_{160}$ and we remove from each of them the same number of elements. These elements that we removed are the elements of M_1. In the second step we repeat the same procedure in the sets that came of the implementation of the first step and so we define M_2. We continue similarly until there are no more elements in $A_1, A_2, ..., A_{160}$, thus defining the sets $M_1, M_2, ..., M_n$. Find the minimum value of n.

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4 Let a triangle ABC inscribed in circle $c(O, R)$ and D an arbitrary point on BC(different from the midpoint).The circumscribed circle of BOD,which is (c_1), meets $c(O, R)$ at K and AB at Z.The circumscribed circle of COD (c_2),meets $c(O, R)$ at M and AC at E.Finally, the circumscribed circle of AEZ (c_3),meets $c(O, R)$ at N.Prove that $\triangle ABC = \triangle KMN$.
