

Greece National Olympiad 2014

www.artofproblemsolving.com/community/c5194

by gavrilos, silouan

- 1 Find all the polynomials with real coefficients which satisfy $(x^2 - 6x + 8)P(x) = (x^2 + 2x)P(x - 2)$ for all $x \in \mathbb{R}$.

- 2 Find all the integers n for which $\frac{8n-25}{n+5}$ is cube of a rational number.

- 3 For even positive integer n we put all numbers $1, 2, \dots, n^2$ into the squares of an $n \times n$ chessboard (each number appears once and only once).
Let S_1 be the sum of the numbers put in the black squares and S_2 be the sum of the numbers put in the white squares. Find all n such that we can achieve $\frac{S_1}{S_2} = \frac{39}{64}$.

- 4 We are given a circle $c(O, R)$ and two points A, B so that $R < AB < 2R$. The circle $c_1(A, r)$ ($0 < r < R$) crosses the circle c at C, D (C belongs to the short arc AB). From B we consider the tangent lines BE, BF to the circle c_1 , in such way that E lays out of the circle c . If $M \equiv EC \cap DF$ show that the quadrilateral $BCFM$ is cyclic.
