4th IGO

www.artofproblemsolving.com/community/c520386

by bgn

– Elementary Level

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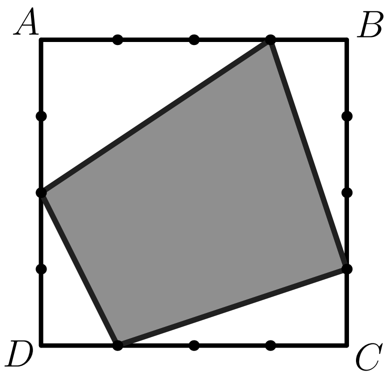
1 Each side of square $ABCD$ with side length of 4 is divided into equal parts by three points. Choose one of the three points from each side, and connect the points consecutively to obtain a quadrilateral. Which numbers can be the area of this quadrilateral? Just write the numbers without proof.



Proposed by Hirad Aalipanah

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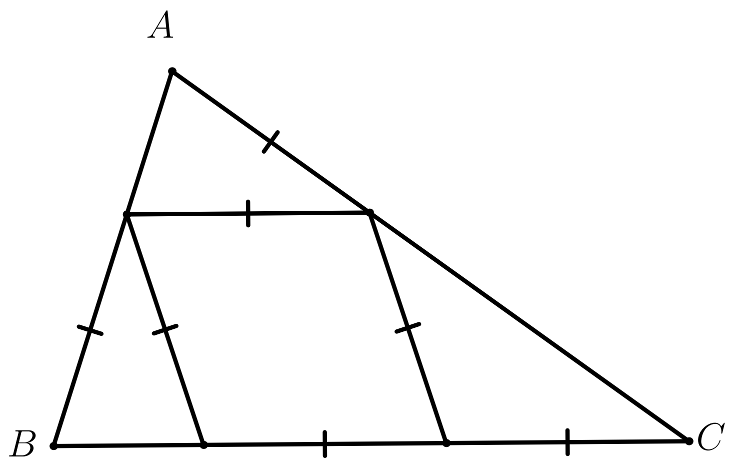
2 Find the angles of triangle ABC.



Proposed by Morteza Saghafian

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3 In the regular pentagon $ABCDE$, the perpendicular at C to CD meets AB at F. Prove that $AE + AF = BE$.

Proposed by Alireza Cheraghi

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4 $P_1, P_2, \ldots, P_{100}$ are 100 points on the plane, no three of them are collinear. For each three points, call their triangle **clockwise** if the increasing order of them is in clockwise order. Can the number of **clockwise** triangles be exactly 2017?

Proposed by Morteza Saghafian

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5 Intermediate level, problem 4 (`https://artofproblemsolving.com/community/u290969h1513413p89968`

– Intermediate Level

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1 Let ABC be an acute-angled triangle with $A = 60^\circ$. Let E, F be the feet of altitudes through B, C respectively. Prove that $CE - BF = \frac{3}{2}(AC - AB)$

Proposed by Fatemeh Sajadi

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2 Two circles ω_1, ω_2 intersect at A, B. An arbitrary line through B meets ω_1, ω_2 at C, D respectively. The points E, F are chosen on ω_1, ω_2 respectively so that $CE = CB,\ BD = DF$. Suppose that BF meets ω_1 at P, and BE meets ω_2 at Q. Prove that A, P, Q are collinear.

Proposed by Iman Maghsoudi

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3 On the plane, n points are given ($n > 2$). No three of them are collinear. Through each two of them the line is drawn, and among the other given points, the one nearest to this line is marked (in each case this point occurred to be unique). What is the maximal possible number of marked points for each given n?

Proposed by Boris Frenkin (Russia)

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4 In the isosceles triangle ABC ($AB = AC$), let l be a line parallel to BC through A. Let D be an arbitrary point on l. Let E, F be the feet of perpendiculars through A to BD, CD respectively. Suppose that P, Q are the images of E, F on l. Prove that $AP + AQ \leq AB$

Proposed by Morteza Saghafian

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5 Let X, Y be two points on the side BC of triangle ABC such that $2XY = BC$ (X is between B, Y). Let AA' be the diameter of the circumcirle of triangle AXY. Let P be the point where AX meets the perpendicular from B to BC, and Q be the point where AY meets the perpendicular from C to BC. Prove that the tangent line from A' to the circumcircle of AXY passes through the circumcenter of triangle APQ.

Proposed by Iman Maghsoudi

– Advanced Level

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1 In triangle ABC, the incircle, with center I, touches the sides BC at point D. Line DI meets AC at X. The tangent line from X to the incircle (different from AC) intersects AB at Y. If YI and BC intersect at point Z, prove that $AB = BZ$.

Proposed by Hooman Fattahimoghaddam

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2 We have six pairwise non-intersecting circles that the radius of each is at least one (no circle lies in the interior of any other circle). Prove that the radius of any circle intersecting all the six circles, is at least one.

Proposed by Mohammad Ali Abam - Morteza Saghafian

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3 Let O be the circumcenter of triangle ABC. Line CO intersects the altitude from A at point K. Let P, M be the midpoints of AK, AC respectively. If PO intersects BC at Y, and the circumcircle of triangle BCM meets AB at X, prove that $BXOY$ is cyclic.

Proposed by Ali Daeinabi - Hamid Pardazi

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4 Three circles $\omega_1, \omega_2, \omega_3$ are tangent to line l at points A, B, C (B lies between A, C) and ω_2 is externally tangent to the other two. Let X, Y be the intersection points of ω_2 with the other common external tangent of ω_1, ω_3. The perpendicular line through B to l meets ω_2 again at Z. Prove that the circle with diameter AC touches ZX, ZY.

Proposed by Iman Maghsoudi - Siamak Ahmadpour

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5 Sphere S touches a plane. Let A, B, C, D be four points on the plane such that no three of them are collinear. Consider the point A' such that S in tangent to the faces of tetrahedron $A'BCD$. Points B', C', D' are defined similarly. Prove that A', B', C', D' are coplanar and the plane $A'B'C'D'$ touches S.

Proposed by Alexey Zaslavsky (Russia)
