Western Mathematical Olympiad 2012

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Day 1

1 Find the smallest positive integer m satisfying the following condition: for all prime numbers p such that $p > 3$,have $105|9^{p^2} - 29^p + m$.
(September 28, 2012, Hohhot)

2 Show that among any $n \geq 3$ vertices of a regular $(2n-1)$-gon we can find 3 of them forming an isosceles triangle.

3 Let A be a set of n elements and $A_1, A_2, ...A_k$ subsets of A such that for any 2 distinct subsets A_i, A_j either they are disjoint or one contains the other. Find the maximum value of k

4 P is a point in the $\triangle\ ABC$,ω is the circumcircle of $\triangle\ ABC$. $BP \cap \omega = \{B, B_1\}$,$CP \cap \omega = \{C, C_1\}$, $PE\ \perp\ AC$,$PF\ \perp\ AB$. The radius of the inscribed circle and circumcircle of ΔABC is r, R respectively. Prove that

$$\frac{EF}{B_1C_1} \geq \frac{r}{R}.$$

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Day 2

1 O is the circumcenter of acute ΔABC, H is the Orthocenter. $AD \perp BC$, EF is the perpendicular bisector of AO,D, E on the BC. Prove that the circumcircle of ΔADE through the midpoint of OH.

2 Define a sequence $\{a_n\}$ by

$$a_0 = \frac{1}{2},\ a_{n+1} = a_n + \frac{a_n^2}{2012}, (n = 0,\ 1,\ 2,\ \cdots),$$

find integer k such that $a_k < 1 < a_{k+1}$.

(September 29, 2012, Hohhot)

3 Let n be a positive integer ≥ 2 . Consider a n by n grid with all entries 1. Define an operation on a square to be changing the signs of all squares adjacent to it but not the sign of its own. Find all n such that it is possible after a finite sequence of operations to reach a n by n grid with all entries -1

4 Find all prime number p, such that there exist an infinite number of positive integer n satisfying the following condition: $p|n^{n+1} + (n+1)^n$.

(September 29, 2012, Hohhot)