Western Mathematical Olympiad 2014

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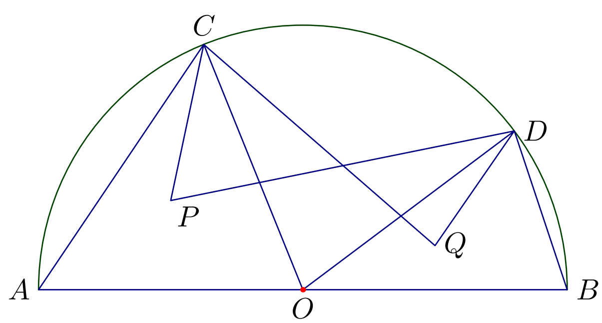
Day 1

1 Let x,y be positive real numbers .Find the minimum of $x+y+\frac{|x-1|}{y}+\frac{|y-1|}{x}$.

2 Let AB be the diameter of semicircle O , C,D be points on the arc AB, P,Q be respectively the circumcenter of $\triangle OAC$ and $\triangle OBD$.
Prove that:$CP\cdot CQ=DP \cdot DQ$.



3 Let $A_1, A_2, ...$ be a sequence of sets such that for any positive integer i, there are only finitely many values of j such that $A_j\subseteq A_i$. Prove that there is a sequence of positive integers $a_1, a_2, ...$ such that for any pair (i, j) to have $a_i \mid a_j \iff A_i \subseteq A_j$.

4 Given a positive integer n, let $a_1,a_2,..,a_n$ be a sequence of nonnegative integers. A sequence of one or more consecutive terms of $a_1,a_2,..,a_n$ is called $dragon$ if their aritmetic mean is larger than 1. If a sequence is a $dragon$, then its first term is the $head$ and the last term is the $tail$. Suppose$a_1,a_2,..,a_n$ is the $head$ or/and $tail$ of some $dragon$ sequence; determine the minimum value of $a_1+a_2+\cdots +a_n$ in terms of n.

Day 2

5 Given a positive integer m, Prove that there exists a positive integers n_0 such that all first digits after the decimal points of $\sqrt{n^2+817n+m}$ in decimal representation are equal, for all integers $n>n_0$.

6 Let $n \geq 2$ is a given integer , $x_1,x_2,\ldots,x_n$ be real numbers such that $(1)x_1+x_2+\ldots+x_n=0$,

$(2)|x_i| \le 1 \; (i = 1, 2, \cdots, n)$.
Find the maximum of Min$\{|x_1 - x_2|, |x_2 - x_3|, \cdots, |x_{n-1} - x_n|\}$.

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7 In the plane, Point O is the center of the equilateral triangle ABC , Points P, Q such that $\overrightarrow{OQ} = 2\overrightarrow{PO}$.
Prove that

$$|PA| + |PB| + |PC| \le |QA| + |QB| + |QC|.$$

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8 Given a real number q, $1 < q < 2$ define a sequence $\{x_n\}$ as follows:
for any positive integer n, let

$$x_n = a_0 + a_1 \cdot 2 + a_2 \cdot 2^2 + \cdots + a_k \cdot 2^k \qquad (a_i \in \{0, 1\}, i = 0, 1, \cdots mk)$$

be its binary representation, define

$$x_k = a_0 + a_1 \cdot q + a_2 \cdot q^2 + \cdots + a_k \cdot q^k.$$

Prove that for any positive integer n, there exists a positive integer m such that $x_n < x_m \le x_n + 1$.

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