China National Olympiad 2009

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Day 1

1	Given an acute triangle PBC with $PB \neq PC$. Points A, D lie on PB, PC, respectively. AC intersects BD at point O. Let E, F be the feet of perpendiculars from O to AB, CD, respectively. Denote by M, N the midpoints of BC, AD. (1): If four points A, B, C, D lie on one circle, then $EM \cdot FN = EN \cdot FM$. (2): Determine whether the converse of (1) is true or not, justify your answer.
2	Find all the pairs of prime numbers (p, q) such that $pq \vert 5^p + 5^q$.
3	Given two integers m, n satisfying $4 < m < n$. Let $A_1A_2 \cdots A_{2n+1}$ be a regular $2n+1$ polygon. Denote by P the set of its vertices. Find the number of convex m polygon whose vertices belongs to P and exactly has two acute angles.

Day 2

1	Given an integer $n > 3$. Let $a_1, a_2, \cdots, a_n$ be real numbers satisfying $min\|a_i - a_j\| = 1, 1 \leq i \leq j \leq n$. Find the minimum value of $\sum_{k=1}^n \|a_k\|^3$.
2	Let P be a convex n polygon each of which sides and diagonals is colored with one of n distinct colors. For which n does: there exists a coloring method such that for any three of n colors, we can always find one triangle whose vertices is of P' and whose sides is colored by the three colors respectively.
3	Given an integer $n > 3$. Prove that there exists a set S consisting of n pairwisely distinct positive integers such that for any two different non-empty subset of S:A, B, $\frac{\sum_{x \in A} x}{\|A\|}$ and $\frac{\sum_{x \in B} x}{\|B\|}$ are two composites which share no common divisors.