South East Mathematical Olympiad 2004

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Day 1

1 Let real numbers a, b, c satisfy $a^2+2b^2+3c^2=\frac{3}{2}$, prove that $3^{-a}+9^{-b}+27^{-c}\geq 1$.

2 In $\triangle$ABC, points D, M lie on side BC and AB respectively, point P lies on segment AD. Line DM intersects segments BP, AC (extended part), PC (extended part) at E, F and N respectively. Show that if DE=DF, then DM=DN.

3 (1) Determine if there exists an infinite sequence $\{a_n\}$ with positive integer terms, such that $a_{n+1}^2\geq 2a_na_{n+2}$ for any positive integer n.
(2) Determine if there exists an infinite sequence $\{a_n\}$ with positive irrational terms, such that $a_{n+1}^2\geq 2a_na_{n+2}$ for any positive integer n.

4 Given a positive integer $n(n>2004)$, we put 1, 2, 3, ,n^2 into squares of an $n\times n$ chessboard with one number in a square. A square is called a good square if the square satisfies following conditions:
1) There are at least 2004 squares that are in the same row with the square such that any number within these 2004 squares is less than the number within the square.
2) There are at least 2004 squares that are in the same column with the square such that any number within these 2004 squares is less than the number within the square.
Find the maximum value of the number of the good square.

Day 2

5 For $\theta\in[0,\frac{\pi}{2}]$, the following inequality $\sqrt{2}(2a+3)\cos(\theta-\frac{\pi}{4})+\frac{6}{\sin\theta+\cos\theta}-2\sin 2\theta<3a+6$ is always true.
Determine the range of a.

6 ABC is an isosceles triangle with AB=AC. Point D lies on side BC. Point F is inside $\triangle$ABC and lies on the circumcircle of triangle ADC. The circumcircle of triangle BDF intersects side AB at point E. Prove that $CD\cdot EF+DF\cdot AE=BD\cdot AF$.

7 A tournament is held among n teams, following such rules:
a) every team plays all others once at home and once away.(i.e. double round-robin schedule)
b) each team may participate in several away games in a week(from Sunday to Saturday).
c) there is no away game arrangement for a team, if it has a home game in the same week.

If the tournament finishes in 4 weeks, determine the maximum value of n.

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8 Determine the number of ordered quadruples (x, y, z, u) of integers, such that

$$\frac{x-y}{x+y}+\frac{y-z}{y+z}+\frac{z-u}{z+u}>0 \text{ and } 1 \leq x, y, z, u \leq 10.$$
