South East Mathematical Olympiad 2009

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by lssl

1 Find all pairs (x, y) of integers such that $x^2 - 2xy + 126y^2 = 2009.$

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2 In the convex pentagon $ABCDE$ we know that $AB = DE, BC = EA$ but $AB \neq EA$. B, C, D, E are concyclic .
Prove that A, B, C, D are concyclic if and only if $AC = AD$.

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3 Let x, y, z be positive reals such that $\sqrt{a} = x(y - z)^2$, $\sqrt{b} = y(z - x)^2$ and $\sqrt{c} = z(x - y)^2$. Prove that

$$a^2 + b^2 + c^2 \geq 2(ab + bc + ca)$$

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4 Given 12 red points on a circle , find the mininum value of n such that there exists n triangles whose vertex are the red points .
Satisfies: every chord whose points are the red points is the edge of one of the n triangles .

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5 Let $X = (x_1, x_2, ..., x_9)$ be a permutation of the set $\{1, 2, \ldots, 9\}$ and let A be the set of all such X .
For any $X \in A$, denote $f(X) = x_1 + 2x_2 + \cdots + 9x_9$ and $M = \{f(X)|X \in A\}$. Find $|M|$. ($|S|$ denotes number of members of the set S.)

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6 Let $\odot O$, $\odot I$ be the circumcircle and inscribed circles of triangleABC . Prove that : From every point D on $\odot O$,we can construct a triangle DEF such that ABC and DEF have the same circumcircle and inscribed circles

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7 Let $x, y, z \geq 0$ be real numbers such that $x + y + z = 1$ Define $f(x, y, z)$ in this way :

$$f(x, y, z) = \frac{x(2y - z)}{1 + x + 3y} + \frac{y(2z - x)}{1 + y + 3z} + \frac{z(2x - y)}{1 + z + 3x}$$

Find the minimum value and maximum value of $f(x, y, z)$.

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8 In an 88 squares chart , we dig out n squares , then we cannot cut a "T"shaped-5-squares out of the surplus chart .
Then find the mininum value of n .
