South East Mathematical Olympiad 2011

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Day 1

1 If $\min\left\{\frac{ax^2+b}{\sqrt{x^2+1}} \mid x\in\mathbb{R}\right\}=3$, then (1) Find the range of b; (2) for every given b, find a.

2 If positive integers, a, b, c are pair-wise co-prime, and,

$$a^2|(b^3+c^3), b^2|(a^3+c^3), c^2|(a^3+b^3)$$

find a, b, and c

3 Find all positive integer n , such that for all 35-element-subsets of $M=(1,2,3,...,50)$,there exists at least two different elements a, b , satisfing : $a-b=n$ or $a+b=n$.

4 Let O be the circumcenter of triangle ABC , a line passes through O intersects sides AB, AC at points M, N , E is the midpoint of MC , F is the midpoint of NB , prove that : $\angle FOE=\angle BAC$

Day 2

1 In triangle ABC , AA_0, BB_0, CC_0 are the angle bisectors , $A_0, B_0, C_0$are on sides BC, CA, AB, draw $A_0A_1//BB_0, A_0A_2//CC_0$,A_1 lies on AC ,A_2 lies on AB , A_1A_2 intersects BC at A_3.B_3 ,C_3 are constructed similarly.Prove that : A_3, B_3, C_3 are collinear.

2 Let P_i $i=1,2,......n$ be n points on the plane , M is a point on segment AB in the same plane , prove : $\sum_{i=1}^{n}|P_iM|\le\max(\sum_{i=1}^{n}|P_iA|, \sum_{i=1}^{n}|P_iB|)$. (Here $|AB|$ means the length of segment AB) .

3 The sequence $(a_n)_{n>=1}$ satisfies that : $a_1=a_2=1$ $a_n=7a_{n-1}-a_{n-2}$ $(n>=3)$, prove that : for all positive integer n , number a_n+2+a_{n+1} is a perfect square .

4 12 points are located on a clock with the sme distance , numbers $1, 2, 3, ...12$ are marked on each point in clockwise order . Use 4 kinds of colors (red,yellow,blue,green) to colour the the points , each kind of color has 3 points . N ow , use these 12 points as the vertex of convex quadrilateral to construct n convex quadrilaterals . They satisfies the following conditions:
(1). the colours of vertex of every convex quadrilateral are different from each other .
(2). for every 3 quadrilaterals among them , there exists a colour such that : the numbers on the 3 points painted into this colour are different from each other .
Find the maximum n .

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