South East Mathematical Olympiad 2012

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Day 1

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1 Find a triple (l,m,n) of positive integers $(1<l<m<n)$, such that $\sum_{k=1}^{l}k$, $\sum_{k=l+1}^{m}k$, $\sum_{k=m+1}^{n}k$ form a geometric sequence in order.

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2 The incircle I of $\triangle ABC$ is tangent to sides AB,BC,CA at D,E,F respectively. Line EF intersects lines AI,BI,DI at M,N,K respectively. Prove that $DM\cdot KE=DN\cdot KF$.

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3 For composite number n, let $f(n)$ denote the sum of the least three divisors of n, and $g(n)$ the sum of the greatest two divisors of n. Find all composite numbers n, such that $g(n)=(f(n))^m$ ($m\in N^*$).

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4 Let a,b,c,d be real numbers satisfying inequality $a\cos x+b\cos 2x+c\cos 3x+d\cos 4x\le 1$ holds for arbitrary real number x. Find the maximal value of $a+b-c+d$ and determine the values of a,b,c,d when that maximum is attained.

Day 2

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1 A nonnegative integer m is called a six-composited number if m and the sum of its digits are both multiples of 6. How many six-composited numbers that are less than 2012 are there?

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2 Find the least natural number n, such that the following inequality holds: $\sqrt{\dfrac{n-2011}{2012}}-\sqrt{\dfrac{n-2012}{2011}}<\sqrt[3]{\dfrac{n-2013}{2011}}-\sqrt[3]{\dfrac{n-2011}{2013}}$.

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3 In $\triangle ABC$, point D lies on side AC such that $\angle ABD=\angle C$. Point E lies on side AB such that $BE=DE$. M is the midpoint of segment CD. Point H is the foot of the perpendicular from A to DE. Given $AH=2-\sqrt{3}$ and $AB=1$, find the size of $\angle AME$.

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4 Let positive integers m,n satisfy $n=2^m-1$. $P_n=\{1,2,\cdots,n\}$ is a set that contains n points on an axis. A grasshopper on the axis can leap from one point to another adjacent point. Find the maximal value of m satisfying following conditions:
(a) x,y are two arbitrary points in P_n;
(b) starting at point x, the grasshopper leaps 2012 times and finishes at point y; (the grasshopper is allowed to travel x and y more than once)
(c) there are even number ways for the grasshopper to do (b).