South East Mathematical Olympiad 2014

www.artofproblemsolving.com/community/c5253

by DraPekka, sqing, parmenides51

–	Grade level 10
Day 1	July 27th
1	Let p be an odd prime.Positive integers a,b,c,d are less than p,and satisfy $p\|a^2+b^2$ and $p\|c^2+d^2$.Prove that exactly one of $ac+bd$ and $ad+bc$ is divisible by p
2	Let $n\geq 4$ be a positive integer.Out of n people,each of two individuals play table tennis game(every game has a winner).Find the minimum value of n,such that for any possible outcome of the game,there always exist an ordered four people group $(a_{1},a_{2},a_{3},a_{4})$,such that the person a_{i} wins against a_{j} for any $1\leq i<j\leq 4$
3	In an obtuse triangle ABC $(AB>AC)$,O is the circumcentre and D,E,F are the midpoints of BC,CA,AB respectively.Median AD intersects OF and OE at M and N respectively.BM meets CN at point P.Prove that $OP\perp AP$
4	Let $x_1,x_2,\cdots,x_n$ be non-negative real numbers such that $x_ix_j\le 4^{-\|i-j\|}$ $(1\le i,j\le n)$. Prove that $$x_1+x_2+\cdots+x_n\le \frac{5}{3}.$$
Day 2	July 28th
5	Let $\triangle ABC$ and $\triangle A'B'C'$ are acute triangles.Prove that $$Max\{cotA'(cotB+cotC),cotB'(cotC+cotA),cotC'(cotA+cotB)\}\geq \frac{2}{3}.$$
6	Let a,b and c be integers and r a real number such that $ar^2+br+c=0$ with $ac\not=0$.Prove that $\sqrt{r^2+c^2}$ is an irrational number
7	Let ω_1 be a circle with centre O. P is a point on ω_1. ω_2 is a circle with centre P, with radius smaller than ω_1. ω_1 meets ω_2 at points T and Q. Let TR be a diameter of ω_2. Draw another two circles with RQ as the radius, R and P as the centres. These two circles meet at point M, with M and Q lie on the same side of PR. A circle with centre M and radius MR intersects ω_2 at R and N. Prove that a circle with centre T and radius TN passes through O.

8 Define a figure which is constructed by unit squares ”cross star” if it satisfies the following conditions: (1)Square bar AB is bisected by square bar CD (2)At least one square of AB lay on both sides of CD (3)At least one square of CD lay on both sides of AB

There is a rectangular grid sheet composed of $38 \times 53 = 2014$ squares,find the number of such cross star in this rectangle sheet

– Grade level 11

Day 1 July 27th

1 Let ABC be a triangle with $AB < AC$ and let M be the midpoint of BC. MI (I incenter) intersects AB at D and CI intersects the circumcircle of ABC at E. Prove that $\frac{ED}{EI} = \frac{IB}{IC}$
https://cdn.artofproblemsolving.com/attachments/0/5/4639d82d183247b875128a842a013ed7415fb
jpg

source (http://artofproblemsolving.com/community/c6h602657p10667541), translated by Antreas Hatzipolakis in fb, corrected by me in order to be compatible with it’s figure

2 Let $n \geq 4$ be a positive integer.Out of n people,each of two individuals play table tennis game(every game has a winner).Find the minimum value of n,such that for any possible outcome of the game,there always exist an ordered four people group (a_1, a_2, a_3, a_4),such that the person a_i wins against a_j for any $1 \leq i < j \leq 4$

3 Let p be a primes ,x, y, z be positive integers such that $x < y < z < p$ and $\{\frac{x^3}{p}\} = \{\frac{y^3}{p}\} = \{\frac{z^3}{p}\}$. Prove that $(x + y + z)|(x^5 + y^5 + z^5)$.

4 Let $x_1, x_2, \cdots, x_n$ be non-negative real numbers such that $x_i x_j \leq 4^{-|i-j|}$ $(1 \leq i, j \leq n)$. Prove that

$$x_1 + x_2 + \cdots + x_n \leq \frac{5}{3}.$$

Day 2 August 28th

5 Let $x_1, x_2, \cdots, x_n$ be positive real numbers such that $x_1 + x_2 + \cdots + x_n = 1$ $(n \geq 2)$. Prove that

$$\sum_{i=1}^{n} \frac{x_i}{x_{i+1} - x_{i+1}^3} \geq \frac{n^3}{n^2 - 1}.$$

here $x_{n+1} = x_1$.

6 Let ω_1 be a circle with centre O. P is a point on ω_1. ω_2 is a circle with centre P, with radius smaller than ω_1. ω_1 meets ω_2 at points T and Q. Let TR be a diameter of ω_2. Draw another two circles with RQ as the radius, R and P as the centres. These two circles meet at point M, with M and Q lie on the same side of PR. A circle with centre M and radius MR intersects ω_2 at R and N. Prove that a circle with centre T and radius TN passes through O.

---

7 Show that there are infinitely many triples of positive integers (a_i, b_i, c_i), $i = 1, 2, 3, \ldots$, satisfying the equation $a^2 + b^2 = c^4$, such that c_n and c_{n+1} are coprime for any positive integer n.

---

8 Define a figure which is constructed by unit squares "cross star" if it satisfies the following conditions: (1)Square bar AB is bisected by square bar CD (2)At least one square of AB lay on both sides of CD (3)At least one square of CD lay on both sides of AB

There is a rectangular grid sheet composed of $38 \times 53 = 2014$ squares,find the number of such cross star in this rectangle sheet

---