Mediterranean Mathematics Olympiad 1999

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1   Do there exist a circle and an infinite set of points on it such that the distance between any two of the points is rational?

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2   A plane figure of area $A > n$ is given, where n is a positive integer. Prove that this figure can be placed onto a Cartesian plane so that it covers at least $n+1$ points with integer coordinates.

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3   Let $a, b, c \neq 0$ and $x, y, z \in \mathbb{R}^+$ such that $x + y + z = 3$. Prove that

$$\frac{3}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}} \geq \frac{x}{1+a^2} + \frac{y}{1+b^2} + \frac{z}{1+c^2}$$

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4   In triangle $\triangle ABC$ we have $BC = a, CA = b, AB = c$ and $\angle B = 4\angle A$ Show that

$$ab^2c^3 = (b^2 - a^2 - ac)((a^2 - b^2)^2 - a^2c^2)$$