

Mediterranean Mathematics Olympiad 2001

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- 1 Let P and Q be points on a circle k . A chord AC of k passes through the midpoint M of PQ . Consider a trapezoid $ABCD$ inscribed in k with $AB \parallel PQ \parallel CD$. Prove that the intersection point X of AD and BC depends only on k and P, Q .

- 2 Find all integers n for which the polynomial $p(x) = x^5 - nx - n - 2$ can be represented as a product of two non-constant polynomials with integer coefficients.

- 3 Show that there exists a positive integer N such that the decimal representation of 2000^N starts with the digits 200120012001.

- 4 Let S be the set of points inside a given equilateral triangle ABC with side 1 or on its boundary. For any $M \in S$, a_M, b_M, c_M denote the distances from M to BC, CA, AB , respectively. Define

$$f(M) = a_M^3(b_M - c_M) + b_M^3(c_M - a_M) + c_M^3(a_M - b_M).$$

- (a) Describe the set $\{M \in S \mid f(M) \geq 0\}$ geometrically.
- (b) Find the minimum and maximum values of $f(M)$ as well as the points in which these are attained.