Mediterranean Mathematics Olympiad 2002

www.artofproblemsolving.com/community/c5258

by , Amir Hossein, MJ GEO

1 Find all natural numbers x, y such that $y|(x^2+1)$ and $x^2|(y^3+1)$.

---

2 Suppose x, y, a are real numbers such that $x+y = x^3+y^3 = x^5+y^5 = a$. Find all possible values of a.

---

3 In an acute-angled triangle ABC, M and N are points on the sides AC and BC respectively, and K the midpoint of MN. The circumcircles of triangles ACN and BCM meet again at a point D. Prove that the line CD contains the circumcenter O of $\triangle ABC$ if and only if K is on the perpendicular bisector of AB.

---

4 If a, b, c are non-negative real numbers with $a^2+b^2+c^2 = 1$, prove that:

$$\frac{a}{b^2+1}+\frac{b}{c^2+1}+\frac{c}{a^2+1} \geq \frac{3}{4}(a\sqrt{a}+b\sqrt{b}+c\sqrt{c})^2$$
