Mediterranean Mathematics Olympiad 2004

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1 Find all natural numbers m such that

$$1! \cdot 3! \cdot 5! \cdots (2m-1)! = \left(\frac{m(m+1)}{2}\right)!.$$

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2 In a triangle ABC, the altitude from A meets the circumcircle again at T. Let O be the circumcenter. The lines OA and OT intersect the side BC at Q and M, respectively. Prove that

$$\frac{S_{AQC}}{S_{CMT}} = \left(\frac{\sin B}{\cos C}\right)^2.$$

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3 Let $a,b,c>0$ and $ab+bc+ca+2abc=1$ then prove that

$$2(a+b+c)+1 \geq 32abc$$

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4 Let z_1, z_2, z_3 be pairwise distinct complex numbers satisfying $|z_1| = |z_2| = |z_3| = 1$ and

$$\frac{1}{2+|z_1+z_2|} + \frac{1}{2+|z_2+z_3|} + \frac{1}{2+|z_3+z_1|} = 1.$$

If the points $A(z_1), B(z_2), C(z_3)$ are vertices of an acute-angled triangle, prove that this triangle is equilateral.
