Mediterranean Mathematics Olympiad 2005

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1 The professor tells Peter the product of two positive integers and Sam their sum. At first, nobody of them knows the number of the other.

One of them says: *You can't possibly guess my number*.
Then the other says: *You are wrong, the number is 136*.

Which number did the professor tell them respectively? Give reasons for your claim.

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2 Let k and k' be concentric circles with center O and radius R and R' where $R < R'$ holds. A line passing through O intersects k at A and k' at B where O is between A and B. Another line passing through O and distict from AB intersects k at E and k' at F where E is between O and F.

Prove that the circumcircles of the triangles OAE and OBF, the circle with diameter EF and the circle with diameter AB are concurrent.

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3 Let $A_1, A_2, \ldots, A_n$ $(n \geq 3)$ be finite sets of positive integers. Prove, that

$$\frac{1}{n}\left(\sum_{i=1}^{n}|A_i|\right) + \frac{1}{\binom{n}{3}}\sum_{1\leq i<j<k\leq n}|A_i \cap A_j \cap A_k| \geq \frac{2}{\binom{n}{2}}\sum_{1\leq i<j\leq n}|A_i \cap A_j|$$

holds, where $|E|$ is the cardinality of the set E

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4 Let A be the set of all polynomials $f(x)$ of order 3 with integer coefficients and cubic coefficient 1, so that for every $f(x)$ there exists a prime number p which does not divide 2004 and a number q which is coprime to p and 2004, so that $f(p) = 2004$ and $f(q) = 0$.
Prove that there exists a infinite subset $B \subset A$, so that the function graphs of the members of B are identical except of translations

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