Mediterranean Mathematics Olympiad 2006

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by WakeUp

1 Every point of a plane is colored red or blue, not all with the same color.
Can this be done in such a way that, on every circumference of radius 1,

(a) there is exactly one blue point;
(b) there are exactly two blue points?

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2 Let P be a point inside a triangle ABC, and A_1B_2, B_1C_2, C_1A_2 be segments passing through P and parallel to AB, BC, CA respectively, where points A_1, A_2 lie on BC, B_1, B_2 on CA, and C_1, C_2 on AB. Prove that

$$\text{Area}(A_1A_2B_1B_2C_1C_2) \geq \frac{1}{2}\text{Area}(ABC)$$

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3 The side lengths a, b, c of a triangle ABC are integers with $\gcd(a, b, c) = 1$. The bisector of angle BAC meets BC at D.

(a) show that if triangles DBA and ABC are similar then c is a square.

(b) If $c = n^2$ is a square $(n \geq 2)$, find a triangle ABC satisfying (a).

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4 Let $0 \leq x_{i,j} \leq 1$, where $i = 1, 2, \ldots m$ and $j = 1, 2, \ldots n$. Prove the inequality

$$\prod_{j=1}^{n}\left(1 - \prod_{i=1}^{m} x_{i,j}\right) + \prod_{i=1}^{m}\left(1 - \prod_{j=1}^{n}(1 - x_{i,j})\right) \geq 1$$

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