

**Mediterranean Mathematics Olympiad 2007**

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1 Let  $x \geq y \geq z$  be real numbers such that  $xy + yz + zx = 1$ . Prove that  $xz < \frac{1}{2}$ . Is it possible to improve the value of constant  $\frac{1}{2}$ ?

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2 The diagonals  $AC$  and  $BD$  of a convex cyclic quadrilateral  $ABCD$  intersect at point  $E$ . Given that  $AB = 39$ ,  $AE = 45$ ,  $AD = 60$  and  $BC = 56$ , determine the length of  $CD$ .

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3 In the triangle  $ABC$ , the angle  $\alpha = \angle BAC$  and the side  $a = BC$  are given. Assume that  $a = \sqrt{rR}$ , where  $r$  is the inradius and  $R$  the circumradius. Compute all possible lengths of sides  $AB$  and  $AC$ .

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4 Let  $x > 1$  be a non-integer number. Prove that

$$\left( \frac{x + \{x\}}{[x]} - \frac{[x]}{x + \{x\}} \right) + \left( \frac{x + [x]}{\{x\}} - \frac{\{x\}}{x + [x]} \right) > \frac{9}{2}$$

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