**Mediterranean Mathematics Olympiad 2011**

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by bluecarneal

**1** A Mediterranean polynomial has only real roots and it is of the form

$$P(x) = x^{10} - 20x^9 + 135x^8 + a_7x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$$

with real coefficients $a_0 \ldots, a_7$. Determine the largest real number that occurs as a root of some Mediterranean polynomial.
*(Proposed by Gerhard Woeginger, Austria)*

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**2** Let $A$ be a finite set of positive reals, let $B = \{x/y \mid x, y \in A\}$ and let $C = \{xy \mid x, y \in A\}$. Show that $|A| \cdot |B| \le |C|^2$.
*(Proposed by Gerhard Woeginger, Austria)*

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**3** A regular tetrahedron of height $h$ has a tetrahedron of height $xh$ cut off by a plane parallel to the base. When the remaining frustrum is placed on one of its slant faces on a horizontal plane, it is just on the point of falling over. (In other words, when the remaining frustrum is placed on one of its slant faces on a horizontal plane, the projection of the center of gravity G of the frustrum is a point of the minor base of this slant face.)
Show that $x$ is a root of the equation $x^3 + x^2 + x = 2$.

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**4** Let $D$ be the foot of the internal bisector of the angle $\angle A$ of the triangle $ABC$. The straight line which joins the incenters of the triangles $ABD$ and $ACD$ cut $AB$ and $AC$ at $M$ and $N$, respectively.
Show that $BN$ and $CM$ meet on the bisector $AD$.

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