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by ilai55, panamath, juckter

– Day 1

1 For every positive integer n let $S(n)$ be the sum of its digits. We say n has a property P if all terms in the infinite secuence $n, S(n), S(S(n)), ...$ are even numbers, and we say n has a property I if all terms in this secuence are odd. Show that for, $1 \leq n \leq 2017$ there are more n that have property I than those who have P.

2 Let ABC be an acute angled triangle and Γ its circumcircle. Led D be a point on segment BC, different from B and C, and let M be the midpoint of AD. The line perpendicular to AB that passes through D intersects AB in E and Γ in F, with point D between E and F. Lines FC and EM intersect at point X. If $\angle DAE = \angle AFE$, show that line AX is tangent to Γ.

3 Consider the configurations of integers $a_{1,1}\, a_{2,1} \quad a_{2,2}\, a_{3,1} \quad a_{3,2} \quad a_{3,3} \cdots \quad \cdots \quad \cdots a_{2017,1} \quad a_{2017,2} \quad a_{2017,3}$
Where $a_{i,j} = a_{i+1,j} + a_{i+1,j+1}$ for all i, j such that $1 \leq j \leq i \leq 2016$.
Determine the maximum amount of odd integers that such configuration can contain.

– Day 2

4 Let ABC be an acute triangle with $AC > AB$ and O its circumcenter. Let D be a point on segment BC such that O lies inside triangle ADC and $\angle DAO + \angle ADB = \angle ADC$. Let P and Q be the circumcenters of triangles ABD and ACD respectively, and let M be the intersection of lines BP and CQ. Show that lines AM, PQ and BC are concurrent.

Pablo Jan, Panama

5 Given a positive integer n, all of its positive integer divisors are written on a board. Two players A and B play the following game:

Each turn, each player colors one of these divisors either red or blue. They may choose whichever color they wish, but they may only color numbers that have not been colored before. The game ends once every number has been colored. A wins if the product of all of the red numbers is a perfect square, or if no number has been colored red, B wins otherwise. If A goes first, determine who has a winning strategy for each n.

6 Let $n > 2$ be an even positive integer and let $a_1 < a_2 < \cdots < a_n$ be real numbers such that $a_{k+1} - a_k \leq 1$ for each $1 \leq k \leq n-1$. Let A be the set of ordered pairs (i,j) with $1 \leq i < j \leq n$ such that $j - i$ is even, and let B the set of ordered pairs (i,j) with $1 \leq i < j \leq n$ such that $j - i$ is odd. Show that

$$\prod_{(i,j)\in A}(a_j - a_i) > \prod_{(i,j)\in B}(a_j - a_i)$$