

Mediterranean Mathematics Olympiad 2013

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by Math-lover123

- 1 Do there exist two real monic polynomials $P(x)$ and $Q(x)$ of degree 3, such that the roots of $P(Q(X))$ are nine pairwise distinct nonnegative integers that add up to 72?
(In a monic polynomial of degree 3, the coefficient of x^3 is 1.)

- 2 Determine the least integer k for which the following story could hold true:
In a chess tournament with 24 players, every pair of players plays at least 2 and at most k games against each other. At the end of the tournament, it turns out that every player has played a different number of games.

- 3 Let x, y, z be positive reals for which: $\sum(xy)^2 = 6xyz$
Prove that: $\sum \sqrt{\frac{x}{x+yz}} \geq \sqrt{3}$.

- 4 $ABCD$ is quadrilateral inscribed in a circle Γ . Lines AB and CD intersect at E and lines AD and BC intersect at F .
Prove that the circle with diameter EF and circle Γ are orthogonal.