Mediterranean Mathematics Olympiad 2014

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by TheMaskedMagician

1 Let $a_1, \ldots, a_n$ and $b_1 \ldots, b_n$ be $2n$ real numbers. Prove that there exists an integer k with $1 \leq k \leq n$ such that $\sum_{i=1}^{n} |a_i - a_k| \quad \leq \quad \sum_{i=1}^{n} |b_i - a_k|$.
(Proposed by Gerhard Woeginger, Austria)

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2 Consider increasing integer sequences with elements from $1, \ldots, 10^6$. Such a sequence is *Adriatic* if its first element equals 1 and if every element is at least twice the preceding element. A sequence is *Tyrrhenian* if its final element equals 10^6 and if every element is strictly greater than the sum of all preceding elements.
Decide whether the number of Adriatic sequences is smaller than, equal to, or greater than the number of Tyrrhenian sequences.
(Proposed by Gerhard Woeginger, Austria)

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3 Prove that for every integer $S \geq 100$ there exists an integer P for which the following story could hold true:
The mathematician asks the shop owner: “How much are the table, the cabinet and the bookshelf?” The shop owner replies: “Each item costs a positive integer amount of Euros. The table is more expensive than the cabinet, and the cabinet is more expensive than the bookshelf. The sum of the three prices is S and their product is P.”
The mathematician thinks and complains: “This is not enough information to determine the three prices!”
(Proposed by Gerhard Woeginger, Austria)

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4 In triangle ABC let A', B', C' respectively be the midpoints of the sides BC, CA, AB. Furthermore let L, M, N be the projections of the orthocenter on the three sides BC, CA, AB, and let k denote the nine-point circle. The lines AA', BB', CC' intersect k in the points D, E, F. The tangent lines on k in D, E, F intersect the lines MN, LN and LM in the points P, Q, R.
Prove that P, Q and R are collinear.
