Moldova Team Selection Test 2010

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Day 1

1 Find all 3-digit numbers such that placing to the right side of the number its successor we get a 6-digit number which is a perfect square.

2 Prove that for any real number x the following inequality is true: $\max\{|\sin x|, |\sin(x+2010)|\} > \dfrac{1}{\sqrt{17}}$

3 Let $ABCD$ be a convex quadrilateral. We have that $\angle BAC = 3\angle CAD$, $AB = CD$, $\angle ACD = \angle CBD$. Find angle $\angle ACD$

4 Let $n \geq 6$ be a even natural number. Prove that any cube can be divided in $\dfrac{3n(n-2)}{4} + 2$ cubes.

Day 2

1 Let $p \in \mathbb{R}_+$ and $k \in \mathbb{R}_+$. The polynomial $F(x) = x^4 + a_3x^3 + a_2x^2 + a_1x + k^4$ with real coefficients has 4 negative roots. Prove that $F(p) \geq (p+k)^4$

2 Let $x_1, x_2, \dots, x_n$ be positive real numbers with sum 1. Find the integer part of: $E = x_1 + \dfrac{x_2}{\sqrt{1-x_1^2}} + \dfrac{x_3}{\sqrt{1-(x_1+x_2)^2}} + \cdots + \dfrac{x_n}{\sqrt{1-(x_1+x_2+\cdots+x_{n-1})^2}}$

3 Let ABC be an acute triangle. H is the orthocenter and M is the middle of the side BC. A line passing through H and perpendicular to HM intersect the segment AB and AC in P and Q. Prove that $MP = MQ$

4 In a chess tournament $2n+3$ players take part. Every two play exactly one match. The schedule is such that no two matches are played at the same time, and each player, after taking part in a match, is free in at least n next (consecutive) matches. Prove that one of the players who play in the opening match will also play in the closing match.