Moldova Team Selection Test 2014

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Day 1 March 3rd

1 Find all pairs of non-negative integers (x, y) such that

$$\sqrt{x+y}-\sqrt{x}-\sqrt{y}+2=0.$$

2 Let $a, b \in \mathbb{R}_+$ such that $a+b=1$. Find the minimum value of the following expression:

$$E(a,b)=3\sqrt{1+2a^2}+2\sqrt{40+9b^2}.$$

3 Let $\triangle ABC$ be an acute triangle and AD the bisector of the angle $\angle BAC$ with $D \in (BC)$. Let E and F denote feet of perpendiculars from D to AB and AC respectively. If $BF \cap CE = K$ and $\odot AKE \cap BF = L$ prove that $DL \perp BF$.

4 Define $p(n)$ to be th product of all non-zero digits of n. For instance $p(5) = 5$, $p(27) = 14$, $p(101) = 1$ and so on. Find the greatest prime divisor of the following expression:

$$p(1)+p(2)+p(3)+...+p(999).$$

Day 2 March 29th

1 Consider $n \geq 2$ positive numbers $0 < x_1 \leq x_2 \leq ... \leq x_n$, such that $x_1 + x_2 + ... + x_n = 1$. Prove that if $x_n \leq \frac{2}{3}$, then there exists a positive integer $1 \leq k \leq n$ such that $\frac{1}{3} \leq x_1+x_2+...+x_k < \frac{2}{3}$.

2 Let a, b, c be positive real numbers such that $abc = 1$. Determine the minimum value of $E(a,b,c) = \sum \frac{a^3+5}{a^3(b+c)}$.

3 Let $ABCD$ be a cyclic quadrilateral. The bisectors of angles BAD and BCD intersect in point K such that $K \in BD$. Let M be the midpoint of BD. A line passing through point C and parallel to AD intersects AM in point P. Prove that triangle $\triangle DPC$ is isosceles.

4 Consider $n \geq 2$ distinct points in the plane $A_1, A_2, ..., A_n$. Color the midpoints of the segments determined by each pair of points in red. What is the minimum number of distinct red points?

Day 3 March 30th

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1 Prove that there do not exist 4 points in the plane such that the distances between any pair of them is an odd integer.

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2 Find all functions $f : R \to R$, which satisfy the equality for any $x, y \in R$: $f(xf(y)+y)+f(xy+x) = f(x+y)+2xy$,

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3 Let $\triangle ABC$ be a triangle with $\angle A$-acute. Let P be a point inside $\triangle ABC$ such that $\angle BAP = \angle ACP$ and $\angle CAP = \angle ABP$. Let M, N be the centers of the incircle of $\triangle ABP$ and $\triangle ACP$, and R the radius of the circumscribed circle of $\triangle AMN$. Prove that $\frac{1}{R} = \frac{1}{AB} + \frac{1}{AC} + \frac{1}{AP}$.

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4 On a circle $n \geq 1$ real numbers are written, their sum is $n-1$. Prove that one can denote these numbers as $x_1, x_2, ..., x_n$ consecutively, starting from a number and moving clockwise, such that for any k ($1 \leq k \leq n$) $x_1 + x_2 + ... + x_k \geq k-1$.

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