**Nordic 2015**

www.artofproblemsolving.com/community/c532722

by parmenides51, rightways

**1** Let $ABC$ be a triangle and $\Gamma$ the circle with diameter $AB$. The bisectors of $\angle BAC$ and $\angle ABC$ intersect $\Gamma$ (also) at $D$ and $E$, respectively. The incircle of $ABC$ meets $BC$ and $AC$ at $F$ and $G$, respectively. Prove that $D, E, F$ and $G$ are collinear.

---

**2** Find the primes $p, q, r$, given that one of the numbers $pqr$ and $p+q+r$ is $101$ times the other.

---

**3** Let $n > 1$ and $p(x) = x^n + a_{n-1}x^{n-1} + ... + a_0$ be a polynomial with $n$ real roots (counted with multiplicity). Let the polynomial $q$ be defined by

$$q(x) = \prod_{j=1}^{2015} p(x+j)$$

.
We know that $p(2015) = 2015$. Prove that $q$ has at least $1970$ different roots $r_1, ..., r_{1970}$ such that $|r_j| < 2015$ for all $j = 1, ..., 1970$.

---

**4** An encyclopedia consists of $2000$ numbered volumes. The volumes are stacked in order with number $1$ on top and $2000$ in the bottom. One may perform two operations with the stack:
(i) For $n$ even, one may take the top $n$ volumes and put them in the bottom of the stack without changing the order.
(ii) For $n$ odd, one may take the top $n$ volumes, turn the order around and put them on top of the stack again.
How many different permutations of the volumes can be obtained by using these two operations repeatedly?

---