Nordic 2003

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1   The squares of a rectangular chessboard with 10 rows and 14 columns are colored alternatingly black and white in the usual manner. Some stones
are placed the board (possibly more than one on the same square) so that there are an odd number of stones in each row and each column.
Show that the total number of stones on black squares is even.

2   Find all triples of integers (x, y, z) satisfying $x^3 + y^3 + z^3 - 3xyz = 2003$

3   The point D inside the equilateral triangle $\triangle ABC$ satisfies $\angle ADC = 150^o$. Prove that a triangle with side lengths $|AD|, |BD|, |CD|$ is necessarily a right-angled triangle.

4   Let $R^* = R - \{0\}$ be the set of non-zero real numbers. Find all functions $f : R^* \to R^*$ satisfying $f(x) + f(y) = f(xyf(x + y))$, for $x, y \in R^*$ and $x + y \neq 0$.