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by parmenides51

1 The trapezium $ABCD$, where AB and CD are parallel and $AD < CD$, is inscribed in the circle c. Let DP be a chord of the circle, parallel to AC. Assume that the tangent to c at D meets the line AB at E and that PB and DC meet at Q. Show that $EQ = AC$.

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2 In two bowls there are in total N balls, numbered from 1 to N. One ball is moved from one of the bowls into the other. The average of the numbers in the bowls is increased in both of the bowls by the same amount, x. Determine the largest possible value of x.

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3 Let $a_1, a_2, ..., a_n,$ and $b_1, b_2, ..., b_n$ be real numbers with $a_1, a_2, ..., a_n$ distinct. Show that if the product $(a_i + b_1)(a_i + b_2)\cdots(a_i + b_n)$ takes the same value for every $i = 1, 2, ..., n,$, then the product $(a_1 + b_j)(a_2 + b_j)\cdots(a_n + b_j)$ also takes the same value for every $j = 1, 2, ..., n,$.

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4 Eva, Per and Anna play with their pocket calculators. They choose different integers and check, whether or not they are divisible by 11. They only look at nine-digit numbers consisting of all the digits $1, 2, ..., 9$. Anna claims that the probability of such a number to be a multiple of 11 is exactly $1/11$. Eva has a different opinion: she thinks the probability is less than $1/11$. Per thinks the probability is more than $1/11$. Who is correct?
