Nordic 2001

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1 Let A be a finite collection of squares in the coordinate plane such that the vertices of all squares that belong to A are $(m, n), (m + 1, n), (m, n + 1)$, and $(m + 1, n + 1)$ for some integers m and n. Show that there exists a subcollection B of A such that B contains at least 25% of the squares in A, but no two of the squares in B have a common vertex.

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2 Let f be a bounded real function defined for all real numbers and satisfying for all real numbers x the condition $f\Big(x+\frac13\Big)+f\Big(x+\frac12\Big)=f(x)+f\Big(x+\frac56\Big)$. Show that f is periodic.

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3 Determine the number of real roots of the equation $x^8 - x^7 + 2x^6 - 2x^5 + 3x^4 - 3x^3 + 4x^2 - 4x + \frac52 = 0$

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4 Let $ABCDEF$ be a convex hexagon, in which each of the diagonals AD, BE , and CF divides the hexagon into two quadrilaterals of equal area. Show that AD, BE , and CF are concurrent.
