

International Mathematical Excellence Olympiad 2017

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by ahmedAbd

- 1 In a game, a player can level up to 16 levels. In each level, the player can upgrade an ability spending that level on it. There are three kinds of abilities, however, one ability can not be upgraded before level 6 for the first time. And that special ability can not be upgraded before level 11. Other abilities can be upgraded at any level, any times (possibly 0), but the special ability needs to be upgraded exactly twice. In how many ways can these abilities be upgraded?

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- 2 Let O be the circumcenter of a triangle ABC . Let M be the midpoint of AO . The BO and CO intersect the altitude AD at points E and F , respectively. Let O_1 and O_2 be the circumcenters of the triangle ABE and ACF , respectively. Prove that M lies on O_1O_2 .

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- 3 A triple (x, y, z) of real numbers is called a *superparticular* if

$$\frac{x+1}{x} \cdot \frac{y+1}{y} = \frac{z+1}{z}$$

Find all superparticular positive integer triples.

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- 4 Let a, b, c be positive real numbers such that $abc = 1$. Prove that

$$\sqrt{\frac{a^3}{1+bc}} + \sqrt{\frac{b^3}{1+ac}} + \sqrt{\frac{c^3}{1+ab}} \geq 2$$

Are there any triples (a, b, c) , for which the equality holds?

Proposed by Konstantinos Metaxas.
