France Team Selection Test 2006

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Day 1

1 Let $ABCD$ be a square and let Γ be the circumcircle of $ABCD$. M is a point of Γ belonging to the arc CD which doesn't contain A. P and R are respectively the intersection points of (AM) with $[BD]$ and $[CD]$, Q and S are respectively the intersection points of (BM) with $[AC]$ and $[DC]$.
Prove that (PS) and (QR) are perpendicular.

2 Let a, b, c be three positive real numbers such that $abc = 1$. Show that:

$$\frac{a}{(a+1)(b+1)} + \frac{b}{(b+1)(c+1)} + \frac{c}{(c+1)(a+1)} \geq \frac{3}{4}.$$

When is there equality?

3 Let a, b be positive integers such that $b^n + n$ is a multiple of $a^n + n$ for all positive integers n. Prove that $a = b$.

Proposed by Mohsen Jamali, Iran

Day 2

1 In a $2 \times n$ array we have positive reals s.t. the sum of the numbers in each of the n columns is 1. Show that we can select a number in each column s.t. the sum of the selected numbers in each row is at most $\frac{n+1}{4}$.

2 Given a triangle ABC satisfying $AC + BC = 3 \cdot AB$. The incircle of triangle ABC has center I and touches the sides BC and CA at the points D and E, respectively. Let K and L be the reflections of the points D and E with respect to I. Prove that the points A, B, K, L lie on one circle.

Proposed by Dimitris Kontogiannis, Greece

3 Let $M = \{1, 2, \ldots, 3 \cdot n\}$. Partition M into three sets A, B, C which $card\ A = card\ B = card\ C = n.$
Prove that there exists a in A, b in B, c in C such that or $a = b + c$, or $b = c + a$, or $c = a + b$

Edited by orl.