Taiwan National Olympiad 1992

www.artofproblemsolving.com/community/c5358

by N.T.TUAN

Day 1

1 Let A, B be two points on a give circle, and M be the midpoint of one of the arcs AB . Point C is the orthogonal projection of B onto the tangent l to the circle at A. The tangent at M to the circle meets AC, BC at A', B' respectively. Prove that if $B\hat{A}C < \frac{\pi}{8}$ then $S_{ABC} < 2S_{A'B'C'}$.

2 Every positive integer can be represented as a sum of one or more consecutive positive integers. For each n , find the number of such representention of n.

3 If $x_1, x_2, ..., x_n (n > 2)$ are positive real numbers with $x_1 + x_2 + ... + x_n = 1$. Prove that $x_1^2x_2 + x_2^2x_3 + ... + x_n^2x_1 \leq \frac{4}{27}$.

Day 2

4 For a positive integer number r, the sequence $a_1, a_2, ...$ defined by $a_1 = 1$ and $a_{n+1} = \frac{na_n+2(n+1)^{2r}}{n+2} \forall n \geq 1$. Prove that each a_n is positive integer number, and find $n's$ for which a_n is even.

5 A line through the incenter I of triangle ABC, perpendicular to AI, intersects AB at P and AC at Q. Prove that the circle tangent to AB at P and to AC at Q is also tangent to the circumcircle of triangle ABC.

6 Find the greatest positive integer A with the following property: For every permutation of $\{1001, 1002, ..., 2000\}$, the sum of some ten consecutive terms is great than or equal to A.