Taiwan National Olympiad 1993

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Day 1

1 A sequence (a_n) of positive integers is given by $a_n=[n+\sqrt{n}+\frac{1}{2}]$. Find all of positive integers which belong to the sequence.

2 Let E and F are distinct points on the diagonal AC of a parallelogram $ABCD$. Prove that , if there exists a cricle through E,F tangent to rays BA,BC then there also exists a cricle through E,F tangent to rays DA,DC.

3 Find all $x,y,z\in\mathbb{N}_{0}$ such that $7^{x}+1=3^{y}+5^{z}$.

Alternative formulation: Solve the equation $1+7^{x}=3^{y}+5^{z}$ in nonnegative integers x, y, z.

Day 2

4 In the Cartesian plane, let C be a unit circle with center at origin O. For any point Q in the plane distinct from O, define Q' to be the intersection of the ray OQ and the circle C. Prove that for any $P\in C$ and any $k\in\mathbb{N}$ there exists a lattice point $Q(x,y)$ with $|x|=k$ or $|y|=k$ such that $PQ'<\frac{1}{2k}$.

5 Assume $A=\{a_{1},a_{2},...,a_{12}\}$ is a set of positive integers such that for each positive integer $n\leq 2500$ there is a subset S of A whose sum of elements is n. If $a_{1}<a_{2}<...<a_{12}$, what is the smallest possible value of a_{1}?

6 Let m be equal to 1 or 2 and $n<10799$ be a positive integer. Determine all such n for which $\sum_{k=1}^{n}\frac{1}{\sin{k}\sin{(k+1)}}=m\frac{\sin{n}}{\sin^{2}{1}}$.