Taiwan National Olympiad 1999

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Day 1

1 Find all triples (x, y, z) of positive integers such that $(x+1)^{y+1}+1=(x+2)^{z+1}$.

2 Let $a_1, a_2, ..., a_{1999}$ be a sequence of nonnegative integers such that for any i, j with $i+j \leq 1999$, $a_i+a_j \leq a_{i+j} \leq a_i+a_j+1$. Prove that there exists a real number x such that $a_n=[nx] \forall n$.

3 There are 1999 people participating in an exhibition. Among any 50 people there are two who don't know each other. Prove that there are 41 people, each of whom knows at most 1958 people.

Day 2

4 Let P^* be the set of primes less than 10000. Find all possible primes $p \in P^*$ such that for each subset $S=\{p_1, p_2, ..., p_k\}$ of P^* with $k \geq 2$ and each $p \notin S$, there is a $q \in P^*-S$ such that $q+1$ divides $(p_1+1)(p_2+1)...(p_k+1)$.

5 Let AD, BE, CF be the altitudes of an acute triangle ABC with $AB>AC$. Line EF meets BC at P, and line through D parallel to EF meets AC and AB at Q and R, respectively. Let N be any poin on side BC such that $\widehat{NQP}+\widehat{NRP}<180^0$. Prove that $BN>CN$.

6 There are eight different symbols designed on $n \geq 2$ different T-shirts. Each shirt contains at least one symbol, and no two shirts contain all the same symbols. Suppose that for any k symbols $(1 \leq k \leq 7)$ the number of shirts containing at least one of the k symbols is even. Determine the value of n.