Lithuania National Olympiad 2006

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1 Solve the system of equations: $\begin{cases} x^4+y^2-xy^3-\frac{9}{8}x=0 \\ y^4+x^2-yx^3-\frac{9}{8}y=0 \end{cases}$

2 Two circles are tangent externaly at a point B. A line tangent to one of the circles at a point A intersects the other circle at points C and D. Show that A is equidistant to the lines BC and BD.

3 Show that if $a+b+c=0$ then $(\frac{a}{b-c}+\frac{b}{c-a}+\frac{c}{a-b})(\frac{b-c}{a}+\frac{c-a}{b}+\frac{a-b}{c})=9$.

4 Find the maximal cardinality $|S|$ of the subset $S \subset A=\{1,2,3,\ldots,9\}$ given that no two sums $a+b|a,b \in S, a \neq b$ are equal.