Lithuania National Olympiad 2010

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– Grade level 10

1 Let a, b be real numbers. Prove the inequality
$$2(a^4+a^2b^2+b^4)\geq 3(a^3b+ab^3).$$

2 In trapezoid $ABCD$, AD is parallel to BC. Knowing that $AB=AD+BC$, prove that the bisector of $\angle A$ also bisects CD.

3 In an $m\times n$ rectangular chessboard,there is a stone in the lower leftmost square. Two persons A,B move the stone alternately. In each step one can move the stone upward or rightward any number of squares. The one who moves it into the upper rightmost square wins. Find all (m,n) such that the first person has a winning strategy.

4 Decimal digits a, b, c satisfy
$$37\mid (a0a0\ldots a0b0c0c\ldots 0c)_{10}$$
where there are 1001 a's and 1001 c's. Prove that $b=a+c$.

– Grade level 11

1 a, b are real numbers such that:
$$a^3+b^3=8-6ab.$$
Find the maximal and minimal value of $a+b$.

2 Let I be the incenter of a triangle ABC. D, E, F are the symmetric points of I with respect to BC, AC, AB respectively. Knowing that D, E, F, B are concyclic,find all possible values of $\angle B$.

3 In an $m\times n$ rectangular chessboard,there is a stone in the lower leftmost square. Two persons A,B move the stone alternately. In each step one can move the stone upward or rightward any number of squares. The one who moves it into the upper rightmost square wins. Find all (m,n) such that the first person has a winning strategy.

4 Arrange arbitrarily $1, 2, \ldots, 25$ on a circumference. We consider all 25 sums obtained by adding 5 consecutive numbers. If the number of distinct residues of those sums modulo 5 is d $(0\leq d\leq 5)$,find all possible values of d.

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