**Team Selection Test For CSMO 2006**

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by carlosbr

**1** Find all the pairs of positive numbers such that the last digit of their sum is 3, their difference is a primer number and their product is a perfect square.

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**2** Let $AA_1$ and $BB_1$ be the altitudes of an acute-angled, non-isosceles triangle $ABC$. Also, let $A_0$ and $B_0$ be the midpoints of its sides $BC$ and $CA$, respectively. The line $A_1B_1$ intersects the line $A_0B_0$ at a point $C'$. Prove that the line $CC'$ is perpendicular to the Euler line of the triangle $ABC$ (this is the line that joins the orthocenter and the circumcenter of the triangle $ABC$).

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**3** The set $M = \{1; 2; 3; \ldots; 29; 30\}$ is divided in $k$ subsets such that if $a + b = n^2, (a, b \in M, a \neq b, n$ is an integer number $)$, then $a$ and $b$ belong different subsets. Determine the minimum value of $k$.

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**4** All the squares of a board of $(n + 1) \times (n - 1)$ squares are painted with **three colors** such that, for any two different columns and any two different rows, the 4 squares in their intersections they don't have all the same color. Find the greatest possible value of $n$.

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